Information Sciences 543 (2021) 112–124

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

Mean-square consensus for heterogeneous multi-agent systems with probabilistic time delay ☆

Fenglan Sun [a,b,c,*], Xiaogang Liao [b], Jürgen Kurths [c,d,e]

[a] Key Lab of Intelligent Analysis and Decision on Complex Systems, School of Science, Chongqing University of Posts and Telecommunications, Chongqing 400065, China
[b] Key Laboratory of Intelligent Air-Ground Cooperative Control for Universities in Chongqing, College of Automation, Chongqing University of Posts and Telecommunications, Chongqing 400065, China
[c] Potsdam Institute for Climate Impact Research, Potsdam 14473, Germany
[d] Institute of Physics, Humboldt University of Berlin, Berlin 12489, Germany
[e] Saratov State University, 83, Astrakhanskaya Street, Saratov 410012, Russia

ARTICLE INFO

Article history:
Received 23 October 2019
Received in revised form 30 May 2020
Accepted 9 July 2020
Available online 23 July 2020

Keywords:
Mean-square consensus
Probabilistic time delay
Multi-agent systems
Heterogeneous dynamics

ABSTRACT

This paper studies the delay-dependent consensus problem of heterogeneous multi-agent systems over directed topology. The heterogeneous dynamics consisting of both first-order and second-order agents with random time delay are considered. New distributed control protocols based on the probability distribution of time delay are proposed for the leader-following and leaderless systems. By adopting matrix theory, Lyapunov-Krasovskii function and stochastic analysis, some less conservative conditions for the mean-square consensus are established over directed fixed topology and switching topologies. Moreover, the larger upper bounds of time delay are obtained. Finally, several simulations are presented to illustrate the obtained results.

1. Introduction

Consensus of multi-agent systems (MASs), which aims to design proper distributed control protocols to make all agents achieve a common value, has attracted great attention due to its wide applications [1–12]. Distributed consensus is generally studied from the two fundamental aspects. The leader-following consensus means that the states of all agents can achieve to that of the leader [4–7]. If there does not exist a specific leader, the study of consensus is called leaderless consensus [8–12]. They both play a crucial role in the applications of MASs.

Most studies on consensus deal with homogeneous systems, in which each agent possesses identical dynamics. However, in practical applications, due to environmental restrictions or different task schedule, agents in the same system often have different dynamics [12–20]. For example, in a robot football match, according to different role tasks, the forward robots and the midfield robots are in charge of shooting, but the full back robots are responsible for the defense of the particular areas.

☆ This work is supported by the National Natural Science Foundation of China (Grant Nos. 61503053, 61673080 and 61773082), the Natural Science Funds of Chongqing CSTC (Grant No. cstc2019jcyj-msxmX0102), the State Scholarship Foundation (Grant No. 201808500022), the Key Project of Crossing and Emerging Area of CQUPT (Grant No. A2018-02), and the project RF Government Grant 075-15-2019-1885.

* Corresponding author at: Key Lab of Intelligent Analysis and Decision on Complex Systems, School of Science, Chongqing University of Posts and Telecommunications, Chongqing 400065, China.

E-mail address: sunfl@cqupt.edu.cn (F. Sun).

https://doi.org/10.1016/j.ins.2020.07.021
0020-0255/

Each robot in a team has a different function and different dynamics. The team is a system with mixed robots, and can be seen as a heterogeneous multi-agent system [20]. Therefore, it is significant to investigate heterogeneous MASs, which mainly refer to systems consisting of both first-order and second-order agents [12–17]. Zheng and Wang [13] analyzed the consensus conditions of heterogeneous MASs over fixed and switching topologies respectively. Consensus of heterogeneous MASs with a leader over switching jointly-connected topologies was investigated in [14]. Guo et al. [15] studied the mean-square consensus with communication noise. Meanwhile, for corresponding discrete-time systems, Zhao and Fei [16] proposed two kinds of control protocols. Kim et al. [17] established sufficient conditions for the consensus with random link failures. Linear and fractional-order heterogeneous MASs were, respectively, investigated in [18,19].

Time delay, an inevitable factor due to the limited transmission conditions, often leads to a poor performance of real systems. For example, for the vehicle–road cooperative system, a vehicle has a reaction time between receiving information from neighbor vehicles and starting to perform tasks. If the reaction time, that is the time delay, is too long, the cooperative control will not be achieved, and even worse, it may lead to a paralyzing of the vehicle–road cooperative system[21]. In the traffic system, time delay may lead to errors in safe driving positioning between vehicles, which will lead to a traffic accident [22]. Therefore, it is very important to study the delay-dependent consensus problem [23–31]. In addition, time delay generally occurs randomly under a dynamically changing environment, especially in vehicle–road cooperative systems [21] and traffic systems [22], where some unexpected accidents may occur. However, the randomness of time delay has not been considered in the aforementioned works, which may lead to conservative results. Besides, the randomness of time delay, such as Bernoulli distribution and Poisson distribution, can often be obtained by statistical methods. Hence, it is practical and meaningful to study the consensus of MASs with probabilistic time delay [32–38]. Ma et al. [32] investigated the consensus problem of Euler–Lagrange systems with probabilistic time delay. Based on a reliable controller, the nonlinear MASs subject to probabilistic time delay was studied in [33]. A consensus protocol for leader-following switched MASs with probabilistic self-delays was proposed in [36]. However, as far as we know, there are no results about the mean-square consensus for heterogeneous MASs with probabilistic time delay till now. Hence, motived by the above discussion, this paper investigates the mean-square consensus for MASs with heterogeneous dynamics and probabilistic time delay. The main contributions of this paper are as follows. (i) The consensus problem for heterogeneous MASs subject to probabilistic time delay is studied, because heterogeneous dynamics and probabilistic time delay are common in real world. (ii) The consensus protocols with probabilistic time delay are designed. (iii) By adopting Lyapunov stability theory and linear matrix inequality (LMI) method, sufficient conditions for the leader-following and leaderless consensus are established. (iv) Based on the established criteria, the larger upper bounds of time delay are obtained.

The rest of this paper is organized as follows. Some basic knowledge and model description are included in Section 2. Main results are discussed in Section 3. Section 4 presents some simulations. And Section 5 gives the conclusions.

2. Preliminaries

Notations: I_n and $\mathbf{0}$, respectively, represent the identity matrix with dimension n and the zero matrix with proper dimension. $\mathbf{1}_n$ and $\mathbf{0}_n$, respectively, are the one and zero column vector with dimension n. $P > 0$ denotes the matrix P being positive definite, $\mathcal{I}_n = \{1, 2, \dots, n\}$ an index set, $\mathrm{B}_1 \otimes \mathrm{B}_2$ the Kronecker product of matrices B_1 and B_2, $*$ the symmetric term in a symmetric matrix, and $\mathbb{E}\{x\}$ the mathematical expectation of x.

The network topology of n agents is modeled by a digraph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{W})$ with the nodes set $\mathcal{V} = \{v_1, v_2, \dots, v_n\}$, the edges set $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ and the adjacency matrix $\mathcal{W} = [w_{ij}] \in \mathbb{R}^{n\times n}$ with $w_{ii} = 0$. $(v_j, v_i) \in \mathcal{E}$ indicates that agent i can directly receive information from agent j. If $(v_j, v_i) \in \mathcal{E}, w_{ij} > 0$, otherwise $w_{ij} = 0$. The neighbor set of agent is denoted by $N_i = \{v_j \in \mathcal{V} | (v_j, v_i) \in \mathcal{E}\}$. And the Laplacian matrix of $\mathcal{G}$ is denoted as $L = [l_{ij}] \in \mathbb{R}^{n\times n}$ with $l_{ij} = -w_{ij}$ for $i \neq j$ and $l_{ii} = \sum_{j=1, j\neq i}^{n} w_{ij}$. Particularly, $L\mathbf{1}_n = \mathbf{0}$. A directed path in $\mathcal{G}$ is a sequence of edges $(v_i, v_{i_{k_1}}), (v_{i_{k_1}}, v_{i_{k_2}}), \dots, (v_{i_{k_l}}, v_j)$ with distinct nodes. The digraph $\mathcal{G}$ contains a directed spanning tree, if there always exists a directed path from one node, which is called the root, to any other node. When node 0 as the leader agent is added, the interaction topology is described by $\bar{\mathcal{G}}$. Then the leader adjacency matrix is denoted by $B = diag\{b_1, b_2, \dots, b_n\} \in \mathbb{R}^{n\times n}$ with $b_i > 0$ if the information of the leader can be obtained by the agent i, otherwise $b_i = 0$.

Consider a system consisting of m $(m < n)$ first-order agents and $n - m$ second-order agents. The dynamics of the first-order agents are described as

$$\dot{p}_i(t) = u_i(t), i \in \mathcal{I}_m, \tag{1}$$

and the dynamics of second-order agents are expressed as

$$\begin{cases} \dot{p}_i(t) = q_i(t), \\ \dot{q}_i(t) = u_i(t), \end{cases} i \in \mathcal{I}_n/\mathcal{I}_m, \tag{2}$$

where $p_i \in \mathbb{R}, q_i \in \mathbb{R}$ and $u_i \in \mathbb{R}$ denote, respectively, the position, velocity and control input of agent i, $\mathcal{I}_m = \{1, 2, \dots, m\}$, correspondingly, $\mathcal{I}_n/\mathcal{I}_m = \{m+1, m+2, \dots, n\}$. The dynamics of the leader labeled as 0 is given as

$$\dot{p}_0 = 0, \tag{3}$$

where $p_0 \in \mathbb{R}$ denotes the position of the leader. The initial states of systems (1)–(2) are denoted as $p(0) = (p_1(0), p_2(0), \ldots, p_n(0))^T$ and $q(0) = (q_{m+1}(0), q_{m+2}(0), \ldots, q_n(0))^T$.

To analyze the probabilistic time delay $h(t)$, a Bernoulli distributed sequence is introduced to describe the occurring process of time delay as

$$\lambda(t) = \begin{cases} 1, & \text{if there no time delay } h(t), \\ 0, & \text{if there exists } h(t), \end{cases}$$

where $\lambda(t)$ satisfies $\text{Prob}\{\lambda(\text{t}) = 1\} = \mathbb{E}\{\lambda(\text{t})\} = \tilde{\lambda}$ and $\text{Prob}\{\lambda(\text{t}) = 0\} = 1 - \mathbb{E}\{\lambda(\text{t})\} = 1 - \tilde{\lambda}$ with the constant $\tilde{\lambda} \in [0, 1]$. Meanwhile, the following assumption, definitions and lemmas are given for further analysis.

Assumption 1. $0 < h(t) \leqslant \tilde{h}$ for $t \geqslant 0$, where the constant $\tilde{h} > 0$.

Remark 1. Based on the probability distribution, probabilistic time delay is introduced, which is more practical in real systems. Moreover, the probabilistic time delay can also be modeled by a Markov process with two models [39,40].

Definition 1. [[15]] Mean-square leader-following consensus for systems (1)–(3) is said to be reached, if for any initial conditions, there is

$$\lim_{t\to\infty} \mathbb{E}||p_i(t) - p_0||^2 = 0, \forall\, i \in \mathcal{I}_n,$$
$$\lim_{t\to\infty} \mathbb{E}||q_i(t)||^2 = 0, \forall\, i \in \mathcal{I}_n/\mathcal{I}_m.$$

Definition 2. [[15]] Mean-square leaderless consensus for systems (1)–(2) is said to be reached, if for any initial conditions, there is

$$\lim_{t\to\infty} \mathbb{E}||p_i(t) - p_j(t)||^2 = 0, \forall\, i,j \in \mathcal{I}_n,$$
$$\lim_{t\to\infty} \mathbb{E}||q_i(t)||^2 = 0, \forall\, i \in \mathcal{I}_n/\mathcal{I}_m.$$

Lemma 1 [41]. *For a matrix $\Psi > 0$ with dimension n and a differentiable vector function $\beta : [0, \mu] \in \mathbb{R}^n$ with the constant $\mu > 0$, the following integral inequality holds*

$$\mu \int_0^\mu \beta^T(v)\Psi\beta(v)dv \geqslant \left(\int_0^\mu \beta(v)dv\right)^T \Psi \left(\int_0^\mu \beta(v)dv\right).$$

Lemma 2. [[42]]*For the symmetric matrix $S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$, where S_{11} and S_{22} are square matrices, then the matrix $S < 0$ is equivalent to $S_{22} < 0$ and $S_{11} - S_{12}S_{22}^{-1}S_{21} < 0$, or $S_{11} < 0$ and $S_{22} - S_{21}S_{11}^{-1}S_{12} < 0$.*

3. Main results

3.1. Leader-following consensus

For systems (1)–(3), design the distributed leader-following consensus protocol as follows

$$u_i(t) = \begin{cases} (1-\lambda(t))k_1\left(\sum_{j=1}^n w_{ij}(p_j(t-h(t)) - p_i(t-h(t))) + b_i(p_0 - p_i(t-h(t)))\right) + \lambda(t)k_1\left(\sum_{j=1}^n w_{ij}(p_j(t) - p_i(t)) + b_i(p_0 - p_i(t))\right), i \in \mathcal{I}_m, \\ (1-\lambda(t))\left(\sum_{j=1}^n w_{ij}(p_j(t-h(t)) - p_i(t-h(t))) + b_i(p_0 - p_i(t-h(t)))\right) + \lambda(t)\left(\sum_{j=1}^n w_{ij}(p_j(t) - p_i(t)) + b_i(p_0 - p_i(t))\right) - k_2 q_i(t), i \in \mathcal{I}_n/\mathcal{I}_m, \end{cases} \tag{4}$$

where $h(t) > 0$ is probabilistic time delay as given in Section 2, the constants $k_1 > 0$ and $k_2 > 0$ are feedback gains.

Let $r(t) = (p_1(t) - p_0, \ldots, p_m(t) - p_0, p_{m+1}(t) - p_0, q_{m+1}(t), \ldots, p_n(t) - p_0, q_n(t))^T$, and

$$L = \begin{bmatrix} L_{ff} & L_{fs} \\ L_{sf} & L_{ss} \end{bmatrix}$$

with

$$L_{ff} = \begin{bmatrix} l_{11} & \cdots & l_{1m} \\ \vdots & \dots & \vdots \\ l_{m1} & \cdots & l_{mm} \end{bmatrix}, \quad L_{fs} = \begin{bmatrix} l_{1(m+1)} & \cdots & l_{1n} \\ \vdots & \dots & \vdots \\ l_{m(m+1)} & \cdots & l_{mn} \end{bmatrix},$$

$$L_{sf} = \begin{bmatrix} l_{(m+1)1} & \cdots & l_{(m+1)m} \\ \vdots & \dots & \vdots \\ l_{n1} & \dots & l_{nm} \end{bmatrix}, \quad L_{ss} = \begin{bmatrix} l_{(m+1)(m+1)} & \cdots & l_{(m+1)n} \\ \vdots & \dots & \vdots \\ l_{n(m+1)} & \cdots & l_{nn} \end{bmatrix},$$

$B = diag\{B_f, B_s\}$ with $B_f = diag\{b_1, b_2, \dots, b_m\}$ and $B_s = diag\{b_{m+1}, b_{m+2}, \dots, b_n\}$.

By combining (1)–(4), the error system of systems (1)–(3) can be written as

$$\dot{r}(t) = (U_1 + \lambda(t)U_2)r(t) + (1 - \lambda(t))U_2 r(t - h(t)), \tag{5}$$

where $U_1 = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & I_{n-m} \otimes \begin{bmatrix} 0 & 1 \\ 0 & -k_2 \end{bmatrix} \end{bmatrix}$, $U_2 = \begin{bmatrix} -k_1(L_{ff} + B_f) & L_{fs} \otimes [\ -k_1 \quad 0\ ] \\ L_{sf} \otimes \begin{bmatrix} 0 \\ -1 \end{bmatrix} & (L_{ss} + B_s) \otimes \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix} \end{bmatrix}$.

Theorem 1. *Let Assumption 1 hold, and let there be a spanning tree in graph $\bar{\mathcal{G}}$ with the leader being the root. Then under the control protocol (4) and the directed topology with $\tilde{h} > 0, k_1 > 0, k_2 > 0, \tilde{\lambda} \in [0, 1]$, the mean-square leader-following consensus for systems (1)–(3) with probabilistic time delay is reached, if there exist matrices $G > 0$ and $E > 0$ with proper dimension such that*

$$\begin{bmatrix} \Lambda_{11} & \Lambda_{12} & (U_1 + U_2)^T E \\ * & \Lambda_{22} & (\tilde{\lambda} - 1)U_2^T E \\ * & * & -\tilde{h}^{-1}E \end{bmatrix} < 0, \tag{6}$$

where $\Lambda_{11} = G(U_1 + U_2) + (U_1 + U_2)^T G, \Lambda_{12} = (\tilde{\lambda} - 1)GU_2$ and $\Lambda_{22} = -\tilde{h}^{-1}E$.

Proof. Construct the Lyapunov–Krasovskii function (LKF) as

$$V(r_t) = V_1(r_t) + V_2(r_t), \tag{7}$$

where

$$V_1(r_t) = r^T(t)Gr(t),$$

$$V_2(r_t) = \int_{t-\tilde{h}}^{t} \int_{s}^{t} \dot{r}^T(v)E\dot{r}(v)dvds,$$

and the symbol r_t is the short form of the function $r(t)$, the same as other similar symbols in the following. Define the infinitesimal operator $\mathcal{L}$ of $V(r_t)$ as

$$\mathcal{L}V(r_t) = \lim_{c \to 0^+} \frac{1}{c}\{\mathbb{E}[V(r_{t+c})|r_t] - V(r_t)\}, \tag{8}$$

Then along (5), similar to the derivation in [32] there is

$$\begin{aligned} \mathcal{L}V_1(r_t) &= 2r^T(t)G[(U_1 + \tilde{\lambda}U_2)r(t) + (\tilde{\lambda} - 1)U_2 r(t - h(t))] \\ &= 2r^T(t)G[(U_1 + U_2)r(t) + (\tilde{\lambda} - 1)U_2 \textstyle\int_{t-h(t)}^{t} \dot{r}(v)dv] \\ &= r^T(t)[(U_1 + U_2)^T G + G(U_1 + U_2)]r(t) + 2(\tilde{\lambda} - 1)r^T(t)GU_2 \textstyle\int_{t-h(t)}^{t} \dot{r}(v)dv, \end{aligned} \tag{9}$$

and

$$\begin{aligned} \mathcal{L}V_2(r_t) &= \tilde{h}\dot{r}^T(t)E\dot{r}(t) - \textstyle\int_{t-\tilde{h}}^{t} \dot{r}^T(v)E\dot{r}(v)dv \\ &= \tilde{h}\eta^T \begin{bmatrix} (U_1 + U_2)^T \\ (\tilde{\lambda} - 1)U_2^T \end{bmatrix} E \begin{bmatrix} (U_1 + U_2)^T \\ (\tilde{\lambda} - 1)U_2^T \end{bmatrix}^T \eta(t) - \textstyle\int_{t-\tilde{h}}^{t} \dot{r}^T(v)E\dot{r}(v)dv, \end{aligned} \tag{10}$$

where $\eta^T(t) = [r^T(t), \int_{t-h(t)}^{t} \dot{r}^T(v)dv]$.

According to Lemma 1, one can obtain

$$\begin{aligned}-\int_{t-\tilde{h}}^{t}\dot{r}^T(v)E\dot{r}(v)dv &\leqslant -\tilde{h}^{-1}\int_{t-\tilde{h}}^{t}\dot{r}^T(v)dvE\int_{t-\tilde{h}}^{t}\dot{r}(v)dv\\ &\leqslant -\tilde{h}^{-1}\int_{t-h(t)}^{t}\dot{r}^T(v)dvE\int_{t-h(t)}^{t}\dot{r}(v)dv.\end{aligned} \tag{11}$$

Then by combining (9)–(11) and taking the expectation for $\mathcal{L}V(r_t)$ there is

$$\mathbb{E}\{\mathcal{L}V(r_t)\} \leqslant \mathbb{E}\{\eta^T(t)\Lambda\eta(t)\}, \tag{12}$$

where $\Lambda = \begin{bmatrix}\Lambda_{11} & \Lambda_{12}\\ * & \Lambda_{22}\end{bmatrix} + \tilde{h}\begin{bmatrix}(U_1+U_2)^T\\ (\tilde{\lambda}-1)U_2^T\end{bmatrix}E\begin{bmatrix}(U_1+U_2)^T\\ (\tilde{\lambda}-1)U_2^T\end{bmatrix}^T$.

By Lemma 2, inequality (6) is equivalent to $\Lambda < 0$. If inequality (6) holds, $\mathbb{E}\{\mathcal{L}V(r_t)\} < 0$. Then one gets that the error system (5) of systems (1)–(3) is mean-square stable, which implies that leader-following systems (1)–(3) achieve the mean-square consensus. This completes the proof.

3.2. Leaderless consensus

For systems (1)–(2), design the following control protocol

$$u_i(t) = \begin{cases}\lambda(t)k_1\sum_{j=1}^{n}w_{ij}(p_j(t)-p_i(t)) + (1-\lambda(t))k_1\sum_{j=1}^{n}w_{ij}(p_j(t-h(t))-p_i(t-h(t))), & i\in\mathcal{I}_m,\\ \lambda(t)\sum_{j=1}^{n}w_{ij}(p_j(t)-p_i(t)) + (1-\lambda(t))\sum_{j=1}^{n}w_{ij}(p_j(t-h(t))-p_i(t-h(t))) - k_2q_i(t), & i\in\mathcal{I}_n/\mathcal{I}_m,\end{cases} \tag{13}$$

where the constants $k_1 > 0$ and $k_2 > 0$ are feedback gains.

Let $y(t) = (p_1(t), p_2(t), \ldots, p_n(t), q_{m+1}(t), q_{m+2}(t), \ldots, q_n(t))^T$. Then under the protocol (13), systems (1)–(2) are equivalently written as

$$\dot{y}(t) = (P_1 + \lambda(t)P_2)y(t) + (1-\lambda(t))P_2y(t-h(t)), \tag{14}$$

where $P_1 = \begin{bmatrix}\mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & I_{n-m}\\ \mathbf{0} & \mathbf{0} & -k_2I_{n-m}\end{bmatrix}$, $P_2 = \begin{bmatrix}-k_1L_{ff} & -k_1L_{fs} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0}\\ -L_{sf} & -L_{ss} & \mathbf{0}\end{bmatrix}$.

Denote $z(t) = (p_2(t)-p_1(t), p_3(t)-p_1(t), \ldots, p_n(t)-p_1(t), q_{m+1}(t), q_{m+2}(t), \ldots, q_n(t))^T$, and

$$M = \begin{bmatrix}-\mathbf{1}_{n-1} & I_{n-1} & \mathbf{0}\\ \mathbf{0}_{n-m} & \mathbf{0} & I_{n-m}\end{bmatrix}, \quad N = \begin{bmatrix}\mathbf{0}_{2n-m-1}^T\\ I_{2n-m-1}\end{bmatrix}.$$

Then there is

$$\begin{cases}z(t) = My(t),\\ y(t) = p_1(t)\begin{bmatrix}\mathbf{1}_n\\ \mathbf{0}_{n-m}\end{bmatrix} + Nz(t).\end{cases} \tag{15}$$

Note that $P_1\begin{bmatrix}\mathbf{1}_n\\ \mathbf{0}_{n-m}\end{bmatrix} = \mathbf{0}$ and $P_2\begin{bmatrix}\mathbf{1}_n\\ \mathbf{0}_{n-m}\end{bmatrix} = \mathbf{0}$. Then the error system can be written as

$$\dot{z}(t) = (J_1 + \lambda(t)J_2)z(t) + (1-\lambda(t))J_2z(t-h(t)), \tag{16}$$

where $J_1 = MP_1N, J_2 = MP_2N$.

Theorem 2. *Let Assumption 1 hold, and let there be a spanning tree in graph $\mathcal{G}$. Then under the control protocol (13) and the directed topology with $\tilde{h} > 0, k_1 > 0, k_2 > 0, \tilde{\lambda} \in [0,1]$, the mean-square leaderless consensus for systems (1)–(2) with probabilistic time delay is reached, if there exist matrices $G > 0$ and $E > 0$ with proper dimension such that*

$$\begin{bmatrix}\chi_{11} & \chi_{12} & (J_1+J_2)^TE\\ * & \chi_{22} & (\tilde{\lambda}-1)J_2^TE\\ * & * & -\tilde{h}^{-1}E\end{bmatrix} < 0, \tag{17}$$

where $\chi_{11} = G(J_1+J_2) + (J_1+J_2)^TG, \chi_{12} = (\tilde{\lambda}-1)GJ_2$, and $\chi_{22} = -\tilde{h}^{-1}E$.

Proof. Define the LKF as

$$V(z_t) = z^T(t)Gz(t) + \int_{t-\tilde{h}}^{t}\int_{s}^{t} \dot{z}^T(v)E\dot{z}(v)dvds. \tag{18}$$

Then along (16) is

$$\begin{aligned} \mathcal{L}V(z_t) \quad & \leqslant z^T(t)[G(J_1+J_2)+(J_1+J_2)^T G]z(t) + 2(\tilde{\lambda}-1)z^T(t)GJ_2\int_{t-h(t)}^{t}\dot{z}(v)dv \\ & +\tilde{h}\bar{\eta}^T(t)\begin{bmatrix}(J_1+J_2)^T\\(\tilde{\lambda}-1)J_2^T\end{bmatrix}E\begin{bmatrix}(J_1+J_2)^T\\(\tilde{\lambda}-1)J_2^T\end{bmatrix}^T\bar{\eta}(t) - \tilde{h}^{-1}\int_{t-h(t)}^{t}\dot{z}^T(v)dvE\int_{t-h(t)}^{t}\dot{z}(v)dv, \end{aligned} \tag{19}$$

where $\bar{\eta}^T(t) = [z^T(t), \int_{t-h(t)}^{t}\dot{z}^T(v)dv]$.

Taking the expectation for (19) one can get

$$\mathbb{E}\{\mathcal{L}V(z_t)\} \leqslant \mathbb{E}\{\bar{\eta}^T(t)\chi\bar{\eta}(t)\}, \tag{20}$$

where $\chi = \begin{bmatrix}\chi_{11} & \chi_{12}\\ * & \chi_{22}\end{bmatrix} + \tilde{h}\begin{bmatrix}(J_1+J_2)^T\\(\tilde{\lambda}-1)J_2^T\end{bmatrix}E\begin{bmatrix}(J_1+J_2)^T\\(\tilde{\lambda}-1)J_2^T\end{bmatrix}^T$.

By Lemma 2, inequality (17) is equivalent to $\chi < 0$. If inequality (17) holds, $\mathbb{E}\{\mathcal{L}V(z_t)\} < 0$. Hence one can obtain that the error system (16) is mean-square stable. Then according to Definition 2, mean-square leaderless consensus for systems (1)–(2) is reached. This completes the proof.

Let the switching signal $\sigma(t) : [0,\infty) \to \aleph = \{1,2,\ldots,\kappa\}$ determine the network graph $\mathcal{G}_\sigma = (\mathcal{V},\mathcal{E}^\sigma,\mathcal{W}^\sigma)$, where $\aleph$ is the index set of network digraphs $\mathcal{G}_\sigma$, positive integer κ is the number of digraphs in the switching topologies. Then under switching topologies $\mathcal{G}_\sigma, \sigma \in \aleph$, design the following consensus protocol

$$u_i(t) = \begin{cases} \lambda(t)k_1\sum_{j=1}^{n}w_{ij}^\sigma(p_j(t)-p_i(t)) + (1-\lambda(t))k_1\sum_{j=1}^{n}w_{ij}^\sigma(p_j(t-h(t))-p_i(t-h(t))), \ i\in\mathcal{I}_m, \\ \lambda(t)\sum_{j=1}^{n}w_{ij}^\sigma(p_j(t)-p_i(t)) + (1-\lambda(t))\sum_{j=1}^{n}w_{ij}^\sigma(p_j(t-h(t))-p_i(t-h(t))) - k_2q_i(t), \ i\in\mathcal{I}_n/\mathcal{I}_m, \end{cases} \tag{21}$$

where the constants $k_1 > 0$ and $k_2 > 0$ are feedback gains.

By taking $z(t) = (p_2(t)-p_1(t), p_3(t)-p_1(t),\ldots,p_n(t)-p_1(t), q_{m+1}(t), q_{m+2}(t),\ldots,q_n(t))^T$ and the similar transformations as (14)–(15), the error system is rewritten as

$$\dot{z}(t) = (J_1 + \lambda(t)J_2^\sigma)z(t) + (1-\lambda(t))J_2^\sigma z(t-h(t)), \tag{22}$$

where $J_2^\sigma = MP_2^\sigma N, P_2 = \begin{bmatrix}-k_1L_{ff}^\sigma & -k_1L_{fs}^\sigma & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0}\\ -L_{sf}^\sigma & -L_{ss}^\sigma & \mathbf{0}\end{bmatrix}, L_{fs}^\sigma, L_{sf}^\sigma, L_{ss}^\sigma$ and L^σ are defined similar to $L_{ff}, L_{fs}, L_{sf}, L_{ss}$ and L with L^σ being the Laplacian matrix of $\mathcal{G}^\sigma$ for $\sigma \in \aleph$.

Theorem 3. *Let Assumption 1 hold, and let there be a spanning tree in graph $\mathcal{G}_\sigma, \sigma \in \aleph$. Then under the control protocol (21) and the directed switching topologies with $\tilde{h} > 0, k_1 > 0, k_2 > 0, \tilde{\lambda} \in [0,1]$, the mean-square leaderless consensus for systems (1)–(2) with probabilistic time delay is reached, if there exist matrices $G > 0$ and $E > 0$ with proper dimension such that*

$$\begin{bmatrix}\partial_{11} & \partial_{12} & (J_1+J_2^\sigma)^T E\\ * & \partial_{22} & (\tilde{\lambda}-1)(J_2{}^\sigma)^T E\\ * & * & -\tilde{h}^{-1}E\end{bmatrix} < 0, \sigma \in \aleph, \tag{23}$$

where $\partial_{11} = G(J_1+J_2^\sigma) + (J_1+J_2^\sigma)^T G, \partial_{12} = (\tilde{\lambda}-1)GJ_2^\sigma$, and $\partial_{22} = -\tilde{h}^{-1}E$.

Proof. Define the LKF as

$$V(z_t) = z^T(t)Gz(t) + \int_{t-\tilde{h}}^{t}\int_{s}^{t} \dot{z}^T(v)E\dot{z}(v)dvds. \tag{24}$$

Then along (22) is

$$\begin{aligned} \mathcal{L}V(z_t) \quad & \leqslant z^T(t)[G(J_1+J_2^\sigma)+(J_1+J_2^\sigma)^T G]z(t) + 2(\tilde{\lambda}-1)z^T(t)GJ_2^\sigma\int_{t-h(t)}^{t}\dot{z}(v)dv \\ & +\tilde{h}\bar{\eta}(t)^T\begin{bmatrix}(J_1+J_2^\sigma)^T\\(\tilde{\lambda}-1)(J_2{}^\sigma)^T\end{bmatrix}E\begin{bmatrix}(J_1+J_2^\sigma)^T\\(\tilde{\lambda}-1)(J_2{}^\sigma)^T\end{bmatrix}^T\bar{\eta}(t) - \tilde{h}^{-1}\int_{t-h(t)}^{t}\dot{z}^T(v)dvE\int_{t-h(t)}^{t}\dot{z}(v)dv, \end{aligned} \tag{25}$$

By taking the expectation for (25), one can get

$$\mathbb{E}\{\mathcal{L}V(z_t)\} \leqslant \mathbb{E}\{\bar{\eta}^T(t)\partial\bar{\eta}(t)\}, \tag{26}$$

where $\partial = \begin{bmatrix} \partial_{11} & \partial_{12} \\ * & \partial_{22} \end{bmatrix} + \tilde{h}\begin{bmatrix} (J_1 + J_2^{\sigma})^T \\ (\tilde{\lambda}-1)(J_2{}^{\sigma})^T \end{bmatrix} E \begin{bmatrix} (J_1 + J_2^{\sigma})^T \\ (\tilde{\lambda}-1)(J_2{}^{\sigma})^T \end{bmatrix}^T$.

By Lemma 2, for any $\sigma \in \aleph$, inequality (23) is equivalent to $\partial < 0$. If inequality (23) holds, $\mathbb{E}\{\mathcal{L}V(z_t)\} < 0$. Hence system (22) is mean-square stable. Then from Definition 2, mean-square leaderless consensus for systems (1)–(2) is reached. This completes the proof.

Remark 2. Similarly, one can also extend the consensus results in Theorem 1 to switching topologies. For the leader-following systems over switching topologies, there should exist a spanning tree in each directed switching topology with the leader being the root.

Remark 3. For simplicity, only one-dimensional systems are considered in the paper. However, the analysis is also valid for high-dimensional systems, and the corresponding results can be obtained by using the properties of matrix Kronecker product.

4. Simulations

For simplicity, suppose that the considered systems in Example 1–3 consist of six agents, which are denoted by node 1–6. There, node 1 and node 2 denote the first-order agents, and nodes 3–6 denote the second-order agents. If there is a leader, it is denoted by node 0. The initial states are given as $p_0 = 40, p(0) = (-15, 15, -20, 20, -25, 20)^T, q(0) = (30, 10, -10, 15)^T$ and the parameters $\tilde{\lambda} = 0.3, k_1 = 1, k_2 = 2$.

Example 1. Consider systems (1)–(3) over the directed topology $\bar{\mathcal{G}}$ as in Fig. 1. By Matlab calculating, the maximum allowable $\tilde{h}$ is obtained as 0.710 when the LMI (6) is solvable with

$$G = \begin{bmatrix}
158.0766 & 0.2950 & 0.6009 & 0.4375 & 0.3528 & 1.6231 & 0.4175 & 3.5101 & -77.0481 & 40.8582 \\
0.2950 & 0.5920 & 1.6442 & 1.0053 & 2.5567 & 1.6369 & 1.6490 & 1.2533 & 0.5250 & 0.5025 \\
0.6009 & 1.6442 & 6.7741 & 3.2977 & 10.5175 & 5.7218 & 6.6974 & 4.2361 & 2.3039 & 1.6515 \\
0.4375 & 1.0053 & 3.2977 & 2.0150 & 5.1560 & 3.2389 & 3.4137 & 2.3256 & 1.0881 & 0.8606 \\
0.3528 & 2.5567 & 10.5175 & 5.1560 & 21.5665 & 9.2773 & 13.8222 & 5.8074 & 5.2828 & 2.1503 \\
1.6231 & 1.6369 & 5.7218 & 3.2389 & 9.2773 & 7.6731 & 7.1872 & 6.7454 & 2.3642 & 2.8012 \\
0.4175 & 1.6490 & 6.6974 & 3.4137 & 13.8222 & 7.18723 & 19.0290 & 5.2844 & 2.5077 & -2.6361 \\
3.5101 & 1.2533 & 4.2361 & 2.3256 & 5.8074 & 6.7454 & 5.2844 & 13.0670 & 4.4180 & 6.6106 \\
-77.0481 & 0.5250 & 2.3039 & 1.0881 & 5.2828 & 2.3642 & 2.5077 & 4.4180 & 47.7280 & -12.6749 \\
40.8582 & 0.5025 & 1.6515 & 0.8606 & 2.15038 & 2.8012 & -2.6361 & 6.6106 & -12.6749 & 23.5474
\end{bmatrix},$$

$$E = \begin{bmatrix}
111.0929 & 0.1720 & 0.3177 & 0.2189 & -1.1132 & 0.6134 & 0.7136 & 1.5297 & -53.94769 & 0.9215 \\
0.1720 & 0.2682 & 0.7962 & 0.4092 & 1.4428 & 0.7666 & 1.1376 & 0.6632 & 0.4139 & 0.2800 \\
0.3177 & 0.7962 & 4.0644 & 1.5925 & 6.3948 & 2.8616 & 4.3238 & 2.2966 & 1.5073 & 0.9779 \\
0.2189 & 0.4092 & 1.5925 & 0.8177 & 2.9709 & 1.5233 & 2.3376 & 1.2715 & 0.8546 & 0.5235 \\
-1.1132 & 1.4428 & 6.3948 & 2.9709 & 14.0356 & 5.0641 & 9.4331 & 3.0989 & 4.0954 & 1.1434 \\
0.6134 & 0.7666 & 2.8616 & 1.5233 & 5.0641 & 3.3353 & 4.4658 & 3.3761 & 1.8476 & 1.4742 \\
0.7136 & 1.1376 & 4.3238 & 2.3376 & 9.4331 & 4.4658 & 11.6845 & 3.0447 & 2.6978 & 0.6651 \\
1.5297 & 0.66327 & 2.2966 & 1.2715 & 3.0989 & 3.3761 & 3.0447 & 5.5039 & 2.6523 & 2.9848 \\
-53.9476 & 0.4139 & 1.5073 & 0.8546 & 4.0954 & 1.8476 & 2.6978 & 2.6523 & 33.7278 & 1.2799 \\
0.9215 & 0.2800 & 0.9779 & 0.5235 & 1.1434 & 1.4742 & 0.6651 & 2.9848 & 1.2799 & 2.3090
\end{bmatrix}.$$

To save space, the solution matrices are omitted in the following simulations. Let the time delay $h(t) = 0.6 + 0.11 \sin t$. Then under the control protocol (4), the position trajectories $p_i(t)$ and the velocity trajectories $q_i(t)$ are given in Figs. 2 and 3, which show that all positions converge to that of the leader and the velocities converge to zero. Furthermore, more information between the maximum allowable $\tilde{h}$ and $1-\tilde{\lambda}$ is given in Table 1, from which one can find that the maximum allowable $\tilde{h}$ increases as the occurrence possibility $1-\tilde{\lambda}$ of the time delay decreases, which verifies the effectiveness and feasibility of the results.

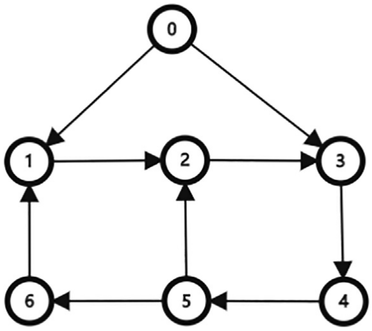

Fig. 1. Network graph $\bar{\mathcal{G}}$ of the leader-following systems.

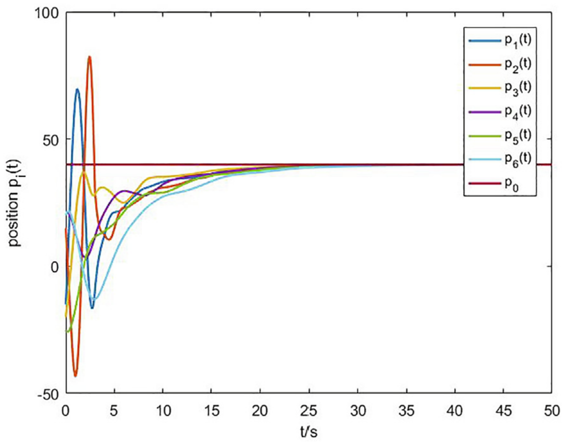

Fig. 2. Position trajectories of all agents.

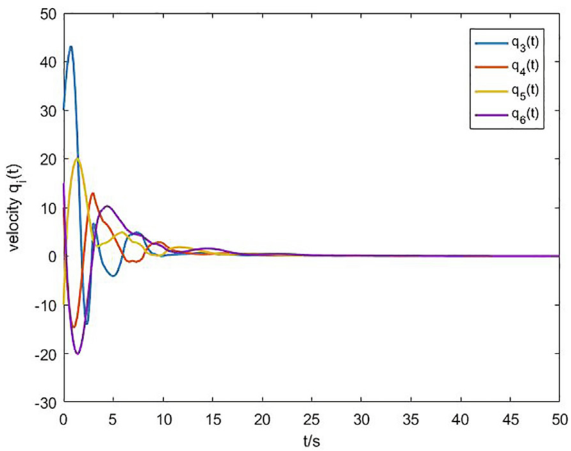

Fig. 3. Velocity trajectories of second-order agents.

Example 2. Consider the leaderless systems (1)–(2) over the directed topology $\mathcal{G}_1$ as in Fig. 4. By Matlab calculating, the maximum allowable $\tilde{h}$ is obtained as 0.545 when the LMI (17) is feasible, which is larger than 0.369 given in [31]. Let the time delay $h(t) = 0.4 + 0.14\sin t$. Then under the control protocol (13), the position trajectories $p_i(t)$ and the velocity trajectories $q_i(t)$ are given in Figs. 5 and 6, which show that all positions converge together and the velocities converge to zero.

Table 1
maximum allowable $\tilde{h}$ for different $1-\tilde{\lambda}$.

$1-\tilde{\lambda}$	0.9	0.7	0.5	0.3	0.1
$\tilde{h}$	0.542	0.710	0.994	1.656	4.970

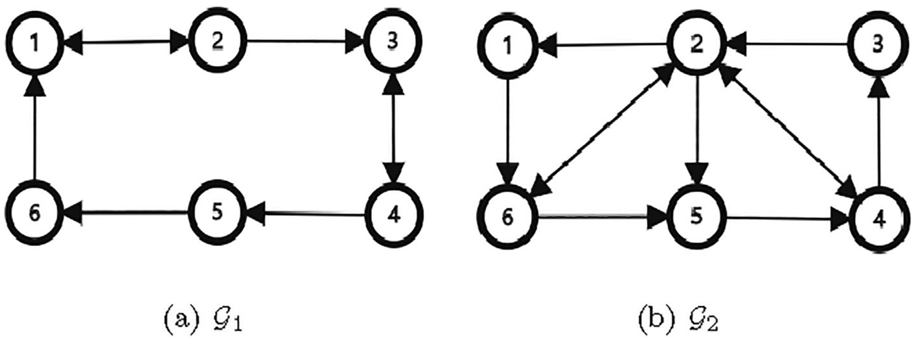

Fig. 4. Network graphs $\mathcal{G}_1$ and $\mathcal{G}_2$ of the leaderless systems.

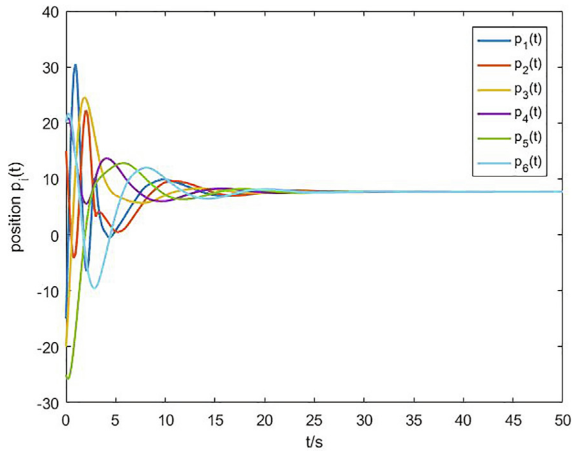

Fig. 5. Position trajectories of all agents.

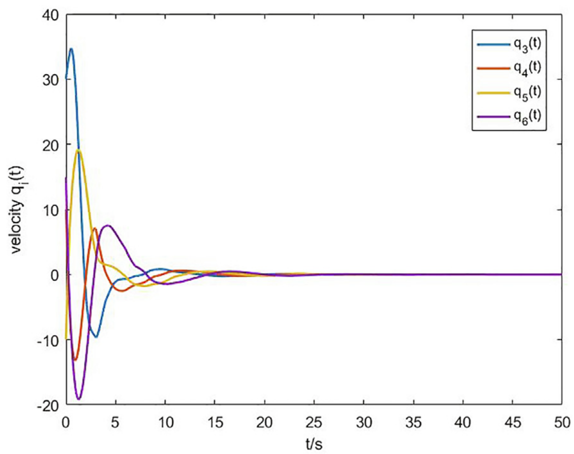

Fig. 6. Velocity trajectories of second-order agents.

Example 3. Consider systems (1)–(2) over the directed switching network graphs $\mathcal{G}_1$ and $\mathcal{G}_2$ as in Fig. 4. By calculations according to Matlab, we obtain that the LMI (23) is feasible for $0 < \tilde{h} \leqslant 0.476$. Choose $h(t) = 0.3 + 0.17\sin t$. Then under the protocol (21), Figs. 7 and 8 show the achievement of the consensus for systems (1)–(2).

Example 4. Note that the established LMIs get bigger as the number of agents grows, this example includes more agents. Consider systems (1)–(2) over the directed topology $\mathcal{G}_3$ as in Fig. 9, where nodes 1–5 denote the first-order agents, and nodes 6–14 the second-order agents. Choose the initial states as $p(0) = (-15, 15, -50, -25, 20, 30, 10, -10, 15, -25, 30, -20, 30, 10)^T$, $q(0) = (20, 40, -40, 30, -30, -10, -20, 40, 30)^T$ and $\tilde{\lambda}, k_1, k_2$ the same as above. By Matlab calculating, the maximum allowable $\tilde{h}$ is obtained as 0.354 when the LMI (17) is feasible. Let $h(t) = 0.2 + 0.15\sin t$. Then under the protocol (13), Figs. 10 and 11 show the achievement of the consensus for systems (1)–(2).

Remark 4. It should be pointed out that the feasibility of the matrix inequalities (6), (17) and (23) is relative to the upper bound and the probability distribution of the time delay. From Table 1, one can find that a larger upper bound could be obtained as the occurrence possibility of the time delay decreases. In Example 2, compared to [31], a larger upper bound of time delay is obtained. Besides, when $\lambda(t) = 0$ always holds, the results in this work are reduced to traditional delay-dependent consensus problem; when $\lambda(t) = 1$ always holds, the results are reduced to traditional delay-less consensus problem. Hence, the obtained results in this paper are more general in comparison with [30,31].

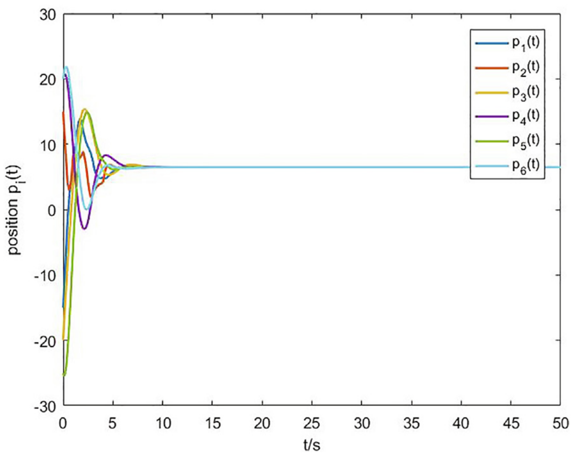

Fig. 7. Position trajectories of all agents.

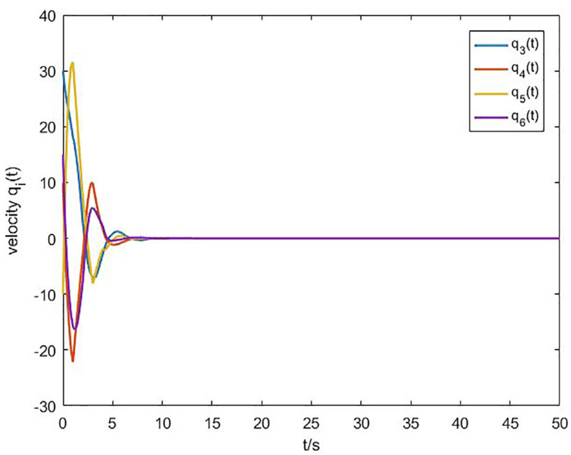

Fig. 8. Velocity trajectories of second-order agents.

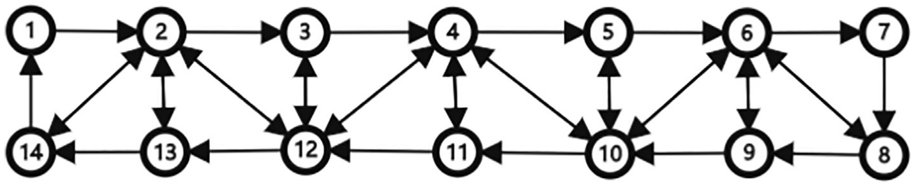

Fig. 9. Network graph $\mathcal{G}_3$ of the leaderless systems.

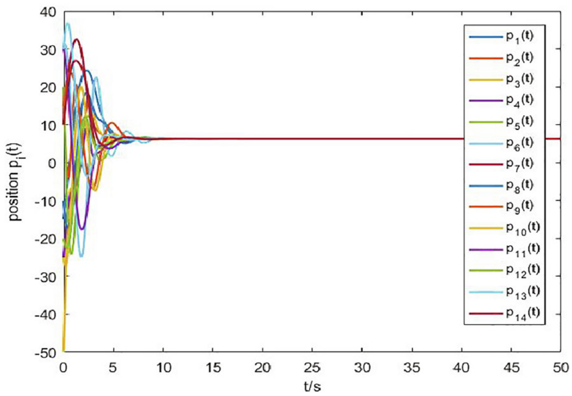

Fig. 10. Position trajectories of all agents.

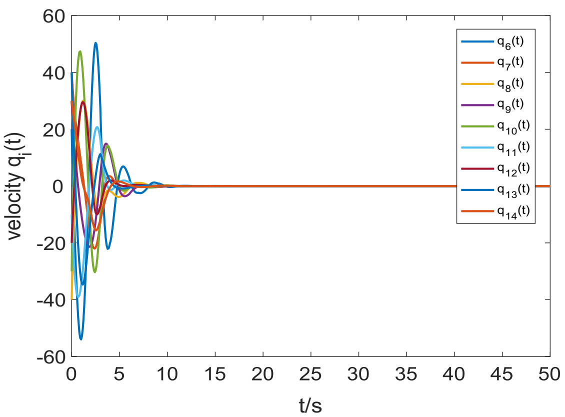

Fig. 11. Velocity trajectories of second-order agents.

Remark 5. It is worth mentioning that the computational complexity of the LMIs indeed grows when the number of agents increases. Fortunately, by adopting the Matlab LMI toolbox, which can be applied to check the feasibility of a LMI, generally, the LMI can be easily solved in a certain time because of the powerful computing capabilities of the computer server and Matlab software.

5. Conclusions

This paper investigates the mean-square consensus problem for heterogeneous MASs with probabilistic time delay. Based on the probability distribution, control theory, and stochastic analysis, some control protocols for the heterogeneous MASs with and without a leader are designed and analysed, and the delay-dependent conditions ensuring the required mean-square consensus are given. Moreover, the larger upper bounds of time delay are obtained by adopting the method in the work. However, the considered leader in this work is static. Due to the complexity of the heterogeneous MASs, a dynamic leader is hard to be treated. In addition, the system over the jointly connected topology, which is more useful in practice, has not been considered. The related problem will be studied in the future work.

CRediT authorship contribution statement

Fenglan Sun: Conceptualization, Methodology, Writing - review & editing, Formal analysis, Funding acquisition, Project administration, Software. **Xiaogang Liao:** Writing-original draft, Validation, Methodology. **Jürgen Kurths:** Supervision, Writing - review & editing, Resources, Software.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgements

The authors would like to sincerely thank the editors and the reviewers for their valuable comments and constructive suggestions, which helped to significantly improve the quality of this paper. This work is supported by the National Natural Science Foundation of China (Grant Nos. 61503053, 61673080 and 61773082), the Natural Science Funds of Chongqing CSTC (Grant No. cstc2019jcyj-msxmX0102), the State Scholarship Foundation (Grant No. 201808500022), the Key Project of Crossing and Emerging Area of CQUPT (Grant No. A2018-02), and the project RF Government Grant 075–15-2019–1885.

References

[1] J. He, P. Cheng, L. Shi, J. Chen, Y. Sun, Time synchronization in WSNs: A maximum-value-based consensus approach, IEEE Trans. Autom. Control 59 (3) (2014) 660–675.
[2] Y. Li, C. Tang, S. Peeta, Y. Wang, Nonlinear consensus based connected vehicle platoon control incorporating car-following interactions and heterogeneous time delays, IEEE Trans. Intell. Transp. Syst. 20 (6) (2019) 2209–2219.
[3] Y. Wang, X. Liu, J. Xiao, Y. Shen, Output formation-containment of interacted heterogeneous linear systems by distributed hybrid active control, Automatica 93 (2018) 26–32.
[4] R. Rakkiyappan, B. Kaviarasan, J. Cao, Leader-following consensus of multi-agent systems via sampled-data control with randomly missing data, Neurocomputing 161 (2015) 132–147.
[5] R. Sakthivel, B. Kaviarasan, C. Ahn, H. Karimi, Observer and stochastic faulty actuator-based reliable consensus protocol for multiagent system, IEEE Trans. Syst., Man, Cybern.: Syst. 48 (12) (2018) 2383–2393.
[6] X. Tan, J. Cao, X. Li, A. Alsaedi, Leader-following mean square consensus of stochastic multi-agent systems with input delay via event-triggered control, IET Control Theory Appl. 12 (2) (2018) 299–309.
[7] C. Deng, G. Yang, Leaderless and leader-following consensus of linear multi-agent systems with distributed event-triggered estimators, J. Franklin Inst. 356 (2019) 309–333.
[8] X. He, Q. Wang, Distributed finite-time leaderless consensus control for double-integrator multi-agent systems with external disturbance, Appl. Math. Comput. 295 (2017) 65–76.
[9] R. Sakthivel, A. Parivallal, B. Kaviarasan, H. Lee, Y. Lim, Finite-time consensus of Markov jumping multi-agent systems with time-varying actuator faults and input saturation, ISA Trans. 83 (2018) 89–99.
[10] X. Tan, J. Cao, L. Rutkowski, G. Lu, Distributed dynamic self-triggered impulsive control for consensus networks: the case of impulse gain with normal distribution, IEEE Trans. Cybern. (2019) 1–11, https://doi.org/10.1109/TCYB.2019.2924258.
[11] R. Sakthivel, R. Sakthivel, B. Kaviarasan, H. Lee, Y. Lim, Finite-time leaderless consensus of uncertain multi-agent systems against time-varying actuator faults, Neurocomputing 325 (2019) 159–171.
[12] J. Bai, G. Wen, A. Rahmani, Leaderless consensus for the fractional-order nonlinear multi-agent systems under directed interaction topology, Int. J. Syst. Sci. 49 (5) (2019) 954–963.
[13] Y. Zheng, L. Wang, Distributed consensus of heterogeneous multi-agent systems with fixed and switching topologies, Int. J. Control 85 (12) (2012) 1967–1976.
[14] L. Mo, Y. Niu, T. Pan, Consensus of heterogeneous multi-agent systems with switching jointly-connected interconnection, Physica A 427 (2015) 132–140.
[15] S. Guo, L. Mo, Y. Yu, Mean-square consensus of heterogeneous multi-agent systems with communication noises, J. Franklin Inst. 355 (8) (2018) 3717–3736.
[16] H. Zhao, S. Fei, Distributed consensus for discrete-time heterogeneous multi-agent systems, Int. J. Control 91 (6) (2018) 1376–1384.
[17] J. Kim, J. Park, Y. Choi, Leaderless and leader-following consensus for heterogeneous multi-agent systems with random link failures, IET Control Theory Appl. 8 (1) (2014) 51–60.
[18] J. Han, H. Zhang, H. Jiang, X. Sun, H_∞ consensus for linear heterogeneous multi-agent systems with state and output feedback control, Neurocomputing 275 (2018) 2635–2644.
[19] X. Yin, D. Yue, S. Hu, Consensus of fractional-order heterogeneous multi-agent systems, IET Control Theory Appl. 7 (2) (2013) 314–322.
[20] Y. Zhou, X. Guan, X. Luo, Finite-time consensus of heterogeneous multi-agent systems with linear and nonlinear dynamics, Acta Automatica Sinica 40 (11) (2014) 2618–2624.
[21] K. Li, Feedback-based formation control for autonomous vehicles under connected environment, Library of Chongqing University of Posts and Telecommunications, Master Thesis, July 2017. .
[22] L. Chai, Clearance coupled operation control method for intelligent vehicle-road cooperative intersection, Library of Beijing Jiaotong University, Ph.D. Thesis, July 2018. .
[23] Y. Chen, Y. Shi, Consensus for linear multiagent systems with time-varying delays: a frequency domain perspective, IEEE Trans. Cybern. 47 (8) (2017) 2143–2150.
[24] A. Proskurnikov, Frequency-domain criteria for consensus in multiagent systems with nonlinear sector-shaped couplings, Autom. Remote Control 75 (1) (2014) 1982–1995.
[25] F. Ye, W. Zhang, L. Ou, H_2 consensus control of time-delayed multi-agent systems: a frequency-domain method, ISA Trans. 66 (2017) 437–447.

[26] J. Hu, Y. Hong, Leader-following coordination of multi-agent systems with coupling time delays, Physica A 374 (2) (2007) 853–863.
[27] P. Yang, Y. Tang, M. Yan, L. Zuo, Leader-follower consensus of second-order multiagent systems with absent velocity measurement and time delay, Math. Problems Eng. 2018 (2018) 1–11.
[28] F. Jiang, L. Wang, G. Xie, Consensus of high-order dynamic multi-agent systems with switching topology and time-varying delays, J. Control Theory Appl. 8 (1) (2010) 52–60.
[29] Y. Sun, L. Wang, Consensus of multi-agent systems in directed networks with nonuniform time-varying delays, IEEE Trans. Autom. Control 54 (7) (2009) 1607–1613.
[30] P. Dai, C. Liu, F. Liu, Consensus problem of heterogeneous multi-agent systems with time delay under fixed and switching topologies, Int. J. Autom. Comput. 11 (3) (2014) 340–346.
[31] B. Wang, Y. Sun, Consensus analysis of heterogeneous multi-agent systems with time-varying delay, Entropy 17 (6) (2015) 3631–3644.
[32] C. Ma, P. Shi, X. Zeng, Q. Zeng, Consensus of Euler-Lagrange systems networked by sampled-data information with probabilistic time delays, IEEE Trans. Cybern. 45 (6) (2014) 1126–1133.
[33] B. Kaviarasan, R. Sakthivel, S. Abbas, Robust consensus of nonlinear multi-agent systems via reliable control with probabilistic time delay, Complexity 21 (S2) (2016) 138–150.
[34] B. Kaviarasan, R. Sakthivel, C. Wang, F. Alzahrani, Resilient control design for consensus of nonlinear multi-agent systems with switching topology and randomly varying communication delays, Neurocomputing 311 (2018) 155–163.
[35] X. Sui, Y. Yang, X. Xu, S. Zhang, L. Zhang, The sampled-data consensus of multi-agent systems with probabilistic time-varying delays and packet losses, Physica A 492 (2018) 1625–1641.
[36] M. Park, O. Kwon, J. Park, S. Lee, E. Cha, A new analysis on leader-following consensus for switched multi-agent systems with time-varying probabilistic self-delays, Int. J. Control Autom. Syst. 13 (3) (2015) 611–619.
[37] X. Tan, J. Cao, L. Rutkowski, Distributed dynamic self-triggered control for uncertain complex networks with Markov switching topologies and random time-varying delay, IEEE Trans. Network Sci. Eng. (2019) 1–9, https://doi.org/10.1109/TNSE.2019.2905758.
[38] J. Wu, Y. Shi, Consensus in multi-agent systems with random delays governed by a Markov chain, Syst. Control Lett. 60 (10) (2011) 863–870.
[39] C. Lin, Z. Wang, F. Yang, Observer-based networked control for continuous-time systems with random sensor delays, Automatica 45 (2) (2009) 578–584.
[40] D. Yue, E. Tian, Z. Wang, J. Lam, Stabilization of systems with probabilistic interval input delays and its applications to networked control systems, IEEE Trans. Syst., Man, Cybern.-Part A: Syst. Humans 39 (4) (2009) 939–945.
[41] K. Gu, J. Chen, V. Kharitonov, Stability of Time-delay Systems, Springer Science & Business Media, 2003.
[42] S. Boyd, L. Ghaoui, E. Feron, V. Balakrishnan, Linear Matrix Inequalities in System and Control Theory, Siam, 1994. .